GCE

Further Mathematics A

Y542/01: Statistics

Advanced GCE

Mark Scheme for June 2019

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This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.

All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

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Annotations and abbreviations

Annotation in scoris	Meaning
✓and ✗	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
Highlighting	
Other abbreviations in mark scheme	**Meaning**
E1	Mark for explaining a result or establishing a given result
dep*	Mark dependent on a previous mark, indicated by *
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working
AG	Answer given
awrt	Anything which rounds to
BC	By Calculator
DR	This question included the instruction: In this question you must show detailed reasoning.

Subject-specific Marking Instructions for AS Level Further Mathematics A

a Annotations should be used whenever appropriate during your marking. The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded. For subsequent marking you must make it clear how you have arrived at the mark you have awarded.

b An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct solutions leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly. Correct but unfamiliar or unexpected methods are often signalled by a correct result following an apparently incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.
If you are in any doubt whatsoever you should contact your Team Leader.

c The following types of marks are available.

M
A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

A
Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B
Mark for a correct result or statement independent of Method marks.

E
Mark for explaining a result or establishing a given result. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

d When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep*' is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

e The abbreviation FT implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only – differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, what is acceptable will be detailed in the mark scheme. If this is not the case please, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.
Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

f Unless units are specifically requested, there is no penalty for wrong or missing units as long as the answer is numerically correct and expressed either in SI or in the units of the question. (e.g. lengths will be assumed to be in metres unless in a particular question all the lengths are in km, when this would be assumed to be the unspecified unit.) We are usually quite flexible about the accuracy to which the final answer is expressed; over-specification is usually only penalised where the scheme explicitly says so. When a value is given in the paper only accept an answer correct to at least as many significant figures as the given value. This rule should be applied to each case. When a value is not given in the paper accept any answer that agrees with the correct value to 2 s.f. Follow through should be used so that only one mark is lost for each distinct accuracy error, except for errors due to premature approximation which should be penalised only once in the examination. There is no penalty for using a wrong value for *g*. E marks will be lost except when results agree to the accuracy required in the question.

g Rules for replaced work: if a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests; if there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others. NB Follow these maths-specific instructions rather than those in the assessor handbook.

h For a genuine misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A mark in the question. Marks designated as cao may be awarded as long as there are no other errors. E marks are lost unless, by chance, the given results are established by equivalent working. 'Fresh starts' will not affect an earlier decision about a misread. Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

i If a calculator is used, some answers may be obtained with little or no working visible. Allow full marks for correct answers (provided, of course, that there is nothing in the wording of the question specifying that analytical methods are required). Where an answer is wrong but there is some evidence of method, allow appropriate method marks. Wrong answers with no supporting method score zero. If in doubt, consult your Team Leader.

j If in any case the scheme operates with considerable unfairness consult your Team Leader.

Question			Answer	Marks	AO	Guidance	
1	(a)		0.8392…	B1 [1]	1.1	Awrt 0.839	S_{xx} = 1.7449…, S_{yy} = 41.2…, S_{xy} = 7.116…
	(b)		$y = -1.180 + 4.0781x$	B1 B1 [2]	1.1 1.1	Both coeffs, awrt –1.18 and 4.08 Letters correct, needs 1 correct coefficient	
	(c)		Value of PMCC suggests that there is strong correlation, *or* 0.75 <u>shown</u> close to mean 0.399	B1 [1]	3.5a	E.g. "*r* high so points lie close to line". "*r* is high" alone is enough. No wrong extras	*Not* "0.75 is close to mean", unless properly justified, e.g. SD (= 0.264) calculated
	(d)		Whether $x = 0.75$ is within the data range	B1 [1]	3.5b	E.g. "maximum and minimum values of *x*"; not "all data points". No wrong extras	Or clear reference to interpolation. NB: 95% CI for *x* is (–0.156, 0.954)
2	(a)		Po(497) P(≥ 520) = 1 – P(≤ 519) used correctly = 0.1564…	B1 M1 A1 [3]	1.1 1.1a 1.1	Stated or implied Allow 0.146(08) from 1 – P(≤ 520) In range [0.156,0.157]	SC: Normal approx.: N(497, 497) B1 In range [0.156, 0.157]: B2
	(b)		Occurrence of a bus is not a random event if it runs on or close to a schedule.	B1 [1]	2.4	Needs context (not just "events"). Allow just "buses not random", or "buses not independent because time between buses is regulated"	*Not* "not independent" without such justification. *Not* "not constant rate". No extras.
3	(a)		B R R B R R B R R B $= \frac{4! \times 6!}{10!} = \frac{24 \times 720}{3628800}$ or $1 \div \frac{10!}{6!4!}$ $= \frac{1}{210}$ or 0.0047619…	M1 M1 A1 [3]	3.1b 1.1a 1.1	This order inferred, e.g. diagram (At least one ! or ${}^nC_r$ or ${}^nP_r$) ÷ 10! *Not* ÷ ${}^{10}C_4$ [except for 1 ÷ ${}^{10}C_4$] Answer, exact or art 0.00476	**Alternatively:** $\frac{4}{10} \times \frac{6}{9} \times \frac{5}{8} \times \frac{3}{7} \times \frac{4}{6} \times \frac{3}{5} \times \frac{2}{4} \times \frac{2}{3} \times \frac{1}{2}$ M1A1 (1 error: M1A0) Answer A1
	(b)		| R | R | R | R | R | R | where the Bs go in a | $= \frac{{}^7C_4 \times 6! \times 4!}{10!} = \frac{35 \times 720 \times 24}{3628800}$ or ${}^7C_4 \div \frac{10!}{6!4!}$ $= \frac{1}{6}$ or 0.1666…	M1 M1 A1 [3]	3.1b 1.1a 1.1	This structure implied, allow 6×| ${}^7C_4$ or ${}^7P_4$ or equivalent used Answer, exact or 0.167 or better	Multiplication: M0M1A0 unless fully correct **Alternatively:** (5 + 20 + 10)/${}^{10}C_4$ (any 3 terms correct): M2. $= \frac{1}{6}$ A1

Question			Answer	Marks	AO	Guidance	
4	(a)		H_0: $\mu = 500$, H_1: $\mu < 500$ where μ is the mean of the greatest weight (that the new design can support)	**B1** **B1**	1.1 2.5	One error, e.g. H_1: $\mu \neq 500$, or μ not defined, or all in words: B1	x or $\bar{x}$: 0 unless defined as population mean (then B1)
			$\bar{X} \sim N(500, \frac{80^2}{40}) = N(500, 160)$ and $\bar{X} = 473$	**M1**	3.3	40 needed but allow √ errors, e.g. variance 80/40 etc. If CV found, *not* centred on 473.	Can be implied by 0.0164, 0.9836, 0.433, 0.198, 0.000 but *not* 0.3679 or 0.00127
			$P(\bar{X} < 473) = 0.01640$ *or* $z = -2.13(45)$ *or* CV = 470.6	**A1**	3.4	p or z correct to 3 sf.	*NOT* 0.9836
			$p > 0.01$ *or* $z > -2.326$ *or* $473 > 470.6$	**A1**	1.1	Compare p with 0.01 or z with –2.326, or 2.326 used in CV	Must be like-with-like, *Not* e.g. $0.9836 > 0.01$ or $p < 2.326$
			Do not reject H_0. Insufficient evidence that greatest weight that new design can support is less than the greatest weight that the traditional design can support.	**M1ft** **A1ft** **[7]**	1.1 2.2b	Correct first conclusion, needs correct method and like-with-like, ft on test statistic if method correct Contextualised, not too definite	But BOD if no explicit comparison of p with 0.01 *Not* "the new design does not have a smaller greatest weight …"
	(b)		Standard deviation/variance remains unchanged, *or* sample must be random	**B1** **[1]**	1.2	No extras. Not "same distribution".	*Not* "assume normal"; this is not needed
	(c)		*Either*: Yes as we do not know that the distribution of weights for the new design is normal *Or*: No as the population distribution known to be normal	**B1** **[1]**	2.1	Allow "population distribution assumed to be normal". No extras, e.g. "and sample size is large".	Allow "yes as we do not know that the distribution for the new design is normal" only if clearly refers to the *new* design only

5	(a)		H_0: no association between orders in races of this type; H_1: positive association between orders	B1 [1]	1.1	Or clear equivalent, e.g. agreement, independent, or H_0: $\rho_s = 0$, H_1: $\rho_s > 0$ *Not* "times"	Allow ρ but not r or r_s unless "population" explicit
	(b)		$1-\frac{6\Sigma d^2}{5\times 24} \geq 0.9$ $\Rightarrow \Sigma d^2 \leq 2$ Largest possible value is 2	M1 M1 A1 [3]	3.1a 1.1a 1.1	Use correct formula and any tabular value of r_s (*not* 0.05 or from r) Solve for Σd^2, needs correct formula Correct conclusion, www but allow = throughout	Allow > or = $\leq$ then $\Sigma d^2 \geq 2$: M1M1A0 Allow from 0.05 Allow $\Sigma d^2 < 2$ so $\Sigma d^2 = 1$ if working correct
	(c)		$\Sigma d^2 = 2 \Rightarrow d = 0, 0, 0, 1, -1$ in some order E.g. *ABCED* or 12354 (allow more than one correct answer)	M1ft M1ft A1 [3]	3.1b 2.2a 2.2b	Turn $\Sigma d^2 \leq$ their 2 into possible values of d Deduce possible order Any order with two consecutive runners interchanged, allow from $\Sigma d^2 = 3$ but no other values	SC: $\Sigma d^2 = 1$, $d = 00001$ M1 1 explicitly impossible A1 Same order only A1 Else ignore ABCDE if seen
SC two-tailed (<u>must be from "H_1: $\rho_s \neq 0$" only</u>): B0; $1 - 6\Sigma d^2/120 \geq 1.0$, $\Sigma d^2 = 0$ M1A1; "same order only" B1, total 3/7							
SC wrong Σd^2: in (c) allow M1M1 for showing any order giving $\Sigma d^2 <$ their (b)							
6	(a)		H_0: no association between city and description of handedness; H_1: some association … Expected frequencies 9.2, 13.8, 30.8, 46.2 $X^2 = 2.0097\ldots+1.3398\ldots+0.6003\ldots+0.4002\ldots$ $= 4.3501\ldots$ > 2.706 Reject H_0. Significant evidence of association between city and description of handedness	B1 M1 M1 A1 A1 M1ft A1ft [7]	2.5 1.1 3.3 3.4 1.1 1.1 2.2b	Or equivalent. Ignore ρ Evidence for correct method Evidence for correct method X^2, 4.35 or better Compare their X^2 with 2.706 Correct first conclusion, needs correct method; contextualised, not too definite	Allow H_0: city and handedness are independent, etc e.g. at least 2 correct values e.g. at least 2 correct values *in same row or column* Allow 2.71 ft on wrong test statistic, *not* on wrong critical value
SC No or wrong Yates (5.42): can get B1M1 M0A0; 5.42 > 2.71 SCB1; M1A1, total 5/7							
	(b)		$P(L \mid A) = P(L)$	B1 [1]	3.3	Or clearly equivalent statement	
	(c)		$P(L \cap A) = 14/100$ $P(A) = 23/100$ so $P(L \mid A) = 14/23$	M1 A1 [2]	3.4 3.5c	$0.14 \div$ any prob or any prob $\div 0.23$ $\frac{14}{23}$ or awrt 0.609	E.g. 14/40 is M1A0

7	(a)		$9 \times \frac{40}{9} = 40$	**B1** **[1]**	1.1	40 or awrt 40.0 only	
	(b)		$\frac{1-p}{p^2} = \frac{40}{9}$	**M1**	2.1	Use correct formula for variance	
			$\Rightarrow p = \frac{3}{8}$ or $-\frac{3}{5}$	**M1** **A1**	3.1b 2.2a	Solve quadratic, can be implied **BC** $\frac{3}{8}$ stated	SC: insufficient working, $\frac{3}{8}$ only: M0B1 for $\frac{3}{8}$, then B0
			Reject $-\frac{3}{5}$	**B1ft**	2.3	Explicitly reject second solution, e.g. "×", no reason needed but not *just* written down and then ignored	Allow for explicit rejection of a solution even if both are wrong
			E(D) = 1/p [$= \frac{8}{3}$]	**M1**	1.1	Use formula for E(D)	
			E(3D + 5) $= 3 \times \frac{8}{3} + 5$ [= 13]	**A1ft** **[6]**	1.1	3×(their E(D)) + 5	p doesn't need to be between 0 and 1 for either of these marks
SC: $\frac{1-p}{p^2} = 40$ (their 40), $p = \frac{-1 \pm \sqrt{161}}{80}$, reject negative solution, E(D) = $\frac{1+\sqrt{161}}{2} = 6.844$, E(3$D$ + 5) = 25.53: M1, M1A0, B1, B2 total 5/6							
	(c)		P(D > E(D)) = P($D \geq 3$)	**M1ft**	3.1a	Convert inequality to integer, their [1/p] + 1, allow >	*Not* their 13 $(1-p)^{8/3}$ [0.286]: M0M1A0
			$= (1-p)^2$	**M1**	1.1a	$(1-p)^r$, ft on their p, r, e.g. 8/3 or 13 Allow $(1-p)^3$ = 125/512 or 0.244	Need $0 < p < 1$ here
			$= \frac{25}{64}$ or 0.390625	**A1** **[3]**	1.1	Answer, exact or art 0.391, www	Allow $(1-p)^6$ = 0.3876 from SC above

8			H_0: $m_Q = m_R$, H_1: $m_Q \neq m_R$, where m_Q and m_R are the medians of the rankings given to Q and R	**B1**	1.1	Allow m undefined. If verbal, must mention medians, m or distribution. Allow $m_d = 0$ as opposed to $m_Q = m_R$	*Not* anything that might be μ unless symbol clearly defined as median. *Not* "there is no difference in the ranks …"
			Sum of ranks = ½ × 54 × 55 = 1485	**M1**	1.1	Find sum of ranks	
			R_m = 1485 – 726 = 759 [or 561]	**A1**	1.1	Correct value of R_m seen	Allow even if 726 used later
			R_m ~ N(660,	**M1**	3.1b	normal, mean their ½ × 24 × 55	
			… 3300)	**A1**	3.3	Both parameters correct	Allow SD/Var muddle
			P(R_m ≥ 759) = 0.0432 (3 s.f.)	**M1**	3.4	Standardise, their R_m	(Same for P(R_m ≤ 561)
			[or z = 1.715]	**A1**	1.1	Correct test statistic (0.0432) 0.0424 or 0.0416 (no/wrong cc): M1A0	Allow $z \in$ [1.71, 1.715], allow z = 1.72 *only* if cc demonstrated correct
			Alternatively:			*Not* 759 – or 726 – …; *not* wrong	
			CV 660 + 1.96√3300 [= 772.6]	**M1**		tail for comparison, but allow ±	Or 561.5 > 547.4
			758.5 < 772.6	**A1**		Needs correct cc	Wrong z-value: M1A1ft B0
			p > 0.025, 2p > 0.05, z < 1.96, or 1.96 used in CV	**B1**	1.1	Explicit correct comparison	Needs like-with-like (e.g. p must be < 0.5)
			Do not reject H_0.	**M1ft**	1.1	Correct first conclusion, needs correct method and like-with-like	ft on wrong ts, or 1-tail/2-tail confusions, e.g. p compared with 0.05 or not explicit, or z ≥ 1.645
			Insufficient evidence of a difference between the ranks.	**A1ft**	2.2b	Contextualised, not too definite	
				[10]			

SC 726 or 594 used: can get B1; M0A0; M1A1; M1A0 [0.1235, 0.1253 or 0.1272]; B1M1A1, total 7/10
For 726 and N(825, 3300) giving p = 0.0432: B1; M0A0; M0A0; M1A1; B1M1A1, total 6/10

9	(a)		Let H(x) be the CDF of $2T$. Then $H(x) = P(X \le x) = P(2T \le x)$ $= P(T \le ½x) = F(½x)$ $= 1 - e^{-0.125x}$ [for $x \ge 0$, and 0 for $x < 0$]	 **M1** **M1** **A1** **[3]**	 3.1a 1.1a 1.1	 Convert to $P(2T \le x)$ Rearrange to get $P(T \le f(x))$ Any letter. Correct answer only, ignore other ranges	**Alternatively:** $g(T) = 2T$, $F(g^{-1}(x))$: M2 Needn't be simplified
	(b)	Due to the error on the paper, all candidates get 7 marks for Q9(b). Annotate each answer with SEEN and enter 7 marks in RM. The only instance where full marks would not be awarded is where a candidate has not attempted any question. This would then need to be a 0.					
			PDF is $f(x) = 0.25e^{-0.25t}$ $E(e^{kt}) = \int_0^\infty 0.25e^{kt}e^{-0.25t}dt$ $\int_0^N 0.25e^{-(0.25-k)t}dt = \left[-\frac{0.25e^{-(0.25-k)t}}{0.25-k}\right]_0^N$ $= -\frac{0.25e^{-(0.25-k)N}}{0.25-k} + \frac{0.25}{0.25-k}$ The first term will only converge for $k < 0.25$ Then $\lim_{N\to\infty} \frac{0.25e^{-(0.25-k)N}}{0.25-k} = 0$ $\int_0^\infty 0.25e^{-(0.25-k)t}dt$ $= \lim_{N\to\infty}\left\{-\frac{0.25e^{-(0.25-k)N}}{0.25-k} + \frac{0.25}{0.25-k}\right\}$ $= \frac{0.25}{0.25-k}$ or $\frac{1}{1-4k}$ **AG**	**M1** **M1** **M1** **A1** **B1** **B1** **A1** **[7]**	2.1 1.1a 2.1 1.1 2.4 2.1 2.2a	Stated or implied Attempt $\int e^{kt}f(t)dt$, correct limits Method for integration Correct indefinite integral with finite upper limit Consider range of k for which the result is valid Correctly obtain given answer	
	(c)		P(no event between 0 and θ) = $P(T > \theta)$ $= e^{-0.25\theta}$ P(0) from $Po(\lambda) = e^{-\lambda}$ Hence same expression, with $\lambda = 0.25\theta$.	**M1** **A1** **B1** **A1** **[4]**	2.1 1.1 1.1 2.2a	Correct method for probability Correct formula Simplified, any λ Correctly justify required result, with $\lambda = 0.25\theta$ oe stated explicitly	$1 - e^{-0.25\theta}$: M1A0 i.e. neither 0! nor e^0 left in Need to say "same" oe

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